Antenna Ratiometry Measurements

For the 21st Century
Using a Homebrew Ratiometer

(*also a Handheld Network Analyzer*)

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Antenna gain measurement by ratiometry has been the preferred and most accurate technique since it was described for the amateur by K2RIW[1] in 1976. However, antenna ratiometry has not been frequently used for amateur antenna ranges in recent years – the probable reason is lack of suitable instrumentation. By taking advantage of some new integrated circuits developed for "wireless" networking, a simple, inexpensive, homebrew ratiometer has been developed for improved antenna measurement. It also functions as a handheld network analyzer with many uses.

Background

Ratiometry measurements use the simultaneous comparison of an unknown quantity. In antenna ratiometry, the comparison is between a TEST antenna and a REFERENCE antenna, side-by-side, illuminated by the same source over similar paths. Any drift or variation in test conditions is seen by both antennas, so the comparison is not affected. As a result, even small differences may be resolved accurately and consistently.

The traditional instrument for antenna ratiometry was the HP-416 Ratiometer, a classic vacuum tube boatanchor. This instrument compared

two channels of detected 1-KHz audio. When hams started using ratiometry for antenna contest ranges, the HP-416 was already long obsolete – I acquired one for $3. It provided good service for many years, but had a number of shortcomings. Like all vacuum tube equipment, it was large and heavy, and drift was a problem. The source had to be AM modulated at a 1 KHz rate, and microwave crystal detectors were used to sense the RF. As a result, dynamic range was limited, with a "magic-eye" tube to indicate that the signals were near the proper level. Readout was on an analog meter, so frequent range switching was required to keep the needle on scale as an antenna was peaked. The biggest problem was that a matched set of microwave crystal detectors was required – a rare and fragile commodity.

Eventually, the antenna measurement responsibility and equipment for the Eastern VHF/UHF Conference was passed on, and the HP-416 stopped appearing. Since then, various other equipment has been used. The only other instruments capable of ratiometric measurements were HP Network Analyzers, kindly lent by Joe Reisert, W1JR, of Antennaco and Dave Olean, K1WHS, of Directive Systems (www.directivesystems.com). For Microwave Update 2002, we were able to borrow a fancy 50 GHz Automatic Network Analyzer from Agilent (www.agilent.com).

A network analyzer is a modern ratiometer, comparing two channels with far greater accuracy and dynamic range than the old HP-416, plus the additional capability for phase measurements. The latest computer-controlled ones are far too expensive for hams, and the older ones, affordable as surplus, are too bulky and too fragile. I have an old HP-8410, the first real network analyzer; if I could even lift it and carry it out for field measurements, I wouldn't expect it to work on arrival.

Lacking a ratiometer or portable network analyzer, I experimented with using a Noise Figure meter, the AIL 75 PANFI. It operates at 30 MHz, so external converters are required. By adding an external switch, it was possible to make good ratiometric measurements[2]. Since the meter is specifically calibrated for Noise Figure measurement, the results required post-processing to get the true ratio in dB. The complete setup, with switch and converters, is still large and bulky.

Homebrew Ratiometer

Since there are no really suitable surplus instruments for antenna ratiometry measurements, is it possible to build something? After pondering this question for a few years, and watching new integrated circuits developed for wireless networking, I discovered a chip with potential, the AD 8302 Gain Phase Detector (www.analog.com). This chip measures the ratio of two channels, with both amplitude and phase outputs, at frequencies up to 2.7 GHz. Sounds a lot like a ratiometer. I acquired a few and tried them out. They worked well over a limited dynamic range, but it wasn't obvious how to stay within the proper range for good, reliable measurements.

I was working with some other new ICs, intended for power measurement, to make a portable power meter[3], when I thought of the "magic-eye" on the old HP416. The best dynamic range for the AD8302 is with a reference channel level of –30 dBm; at this level, the test channel range is ± 30 dB, more than adequate for antenna measurements. A power measurement IC could be used to keep the reference channel within a few dB of –30 dBm, so we could trust any reading within ± 20 dB. This is probably sufficient for any amateur antenna range – an antenna with gain more than 20 dB higher than the reference antenna is probably too big for the range, and more than 20 dB lower is probably defective.

Now we have a strategy: monitor the reference power, keep it near –30 dBm, and display ± 20 dB for the ratio measurement. I chose a digital panel meter for the display: a ± 2 volt range, with the proper decimal point inserted, would cover the full range with excellent precision. A simple op-amp circuit amplifies the AD8302 output so that 20 dB = 2 volts. Another section of a quad op-amp inverts the phase output so it reads directly in degrees, and a third section shifts the return side of the digital power meter to adjust the meter zero.

For the power measurement IC, I experimented with several and settled on the LT5534 (www.linear.com) for wide dynamic range over the full frequency range. This chip detects powers lower than –50 dBm and is quite linear up to about –10 dBm. The output goes to a pair of comparators: one lights a green LED at –40 dBm, about the minimum power needed for usable readings, and the other lights a yellow LED at –20 dBm, about the maximum power before the maximum reading is compressed. The power detector output also goes to an LED bargraph indicator to operate as a "magic-eye", using a circuit from WW2R[4]. I set the levels so that when half the bars are lit, the power is within 3 dB of –30 dBm.

An extra quad comparator section is wired as a peak-hold circuit, to make it easy to find the maximum gain.

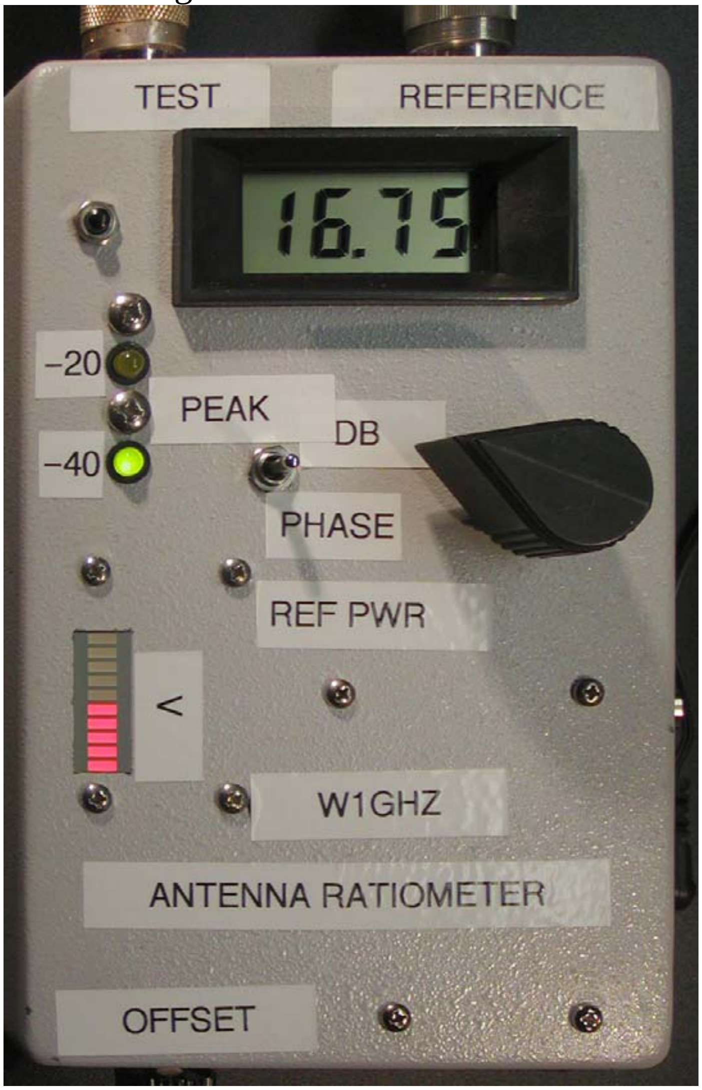


Figure 1

The completed antenna ratiometer is shown in Figure 1. The diecast aluminum box is small enough to be handheld, and the LCD digital power meter display is large enough to be read easily in any lighting. The rotary switch selects between ratio in dB, phase, and the REFERENCE power detector output. A toggle switch selects the peak-hold reading, and a push button clears the peak by discharging a capacitor. At one end of the box is the zero or offset knob, and the other has coax connectors for the two input channels. On one side are a power connector and a ratio output, intended to drive a Tonemeter[5] to enable antenna pointing by ear.

Inside the box, Figure 2, are a circuit board containing the ratiometer circuit, a small board containing the LED bargraph circuit, and the digital power meter. Most inexpensive digital power meters must be isolated from the circuit being measured, so that a separate 9-volt battery would be required, but the stock no. 14505 ME from www.mpja.com is capable of being connected to the circuit being measured, so a separate battery is not needed. The schematic of the ratiometer circuit board is shown in Figure 4, and the box wiring is shown in Figure 5. For the LED bargraph circuit, I would recommend the RFPM from Down East Microwave (www.downeastmicrowave.com); I would have used one, but the diecast box already had a cutout for the LED bargraph and the RFPM board didn't fit, so I made my own smaller board.

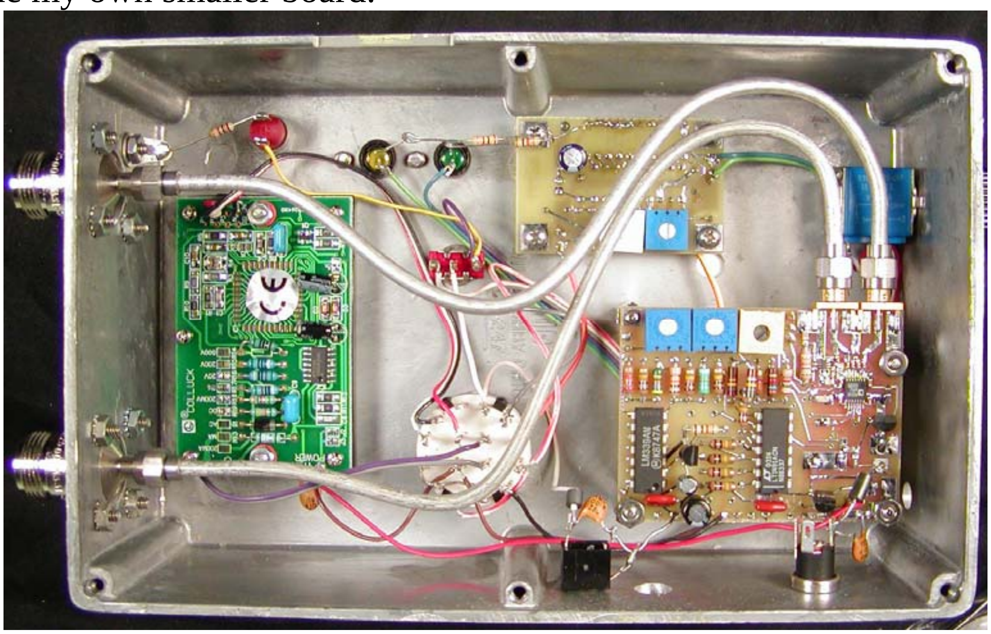


Figure 2

A closeup of the RF section, with the AD8302 and the tiny LT5534, is shown in Figure 3. All the chip components are the very small 0603 size except for C14, a 1µf capacitor.

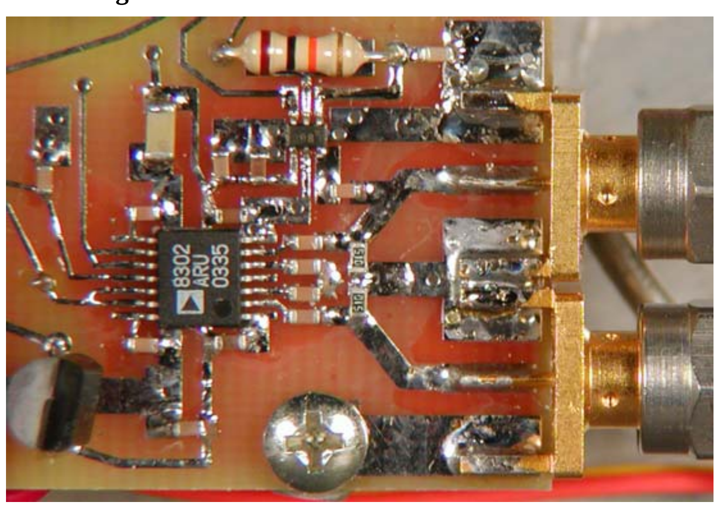
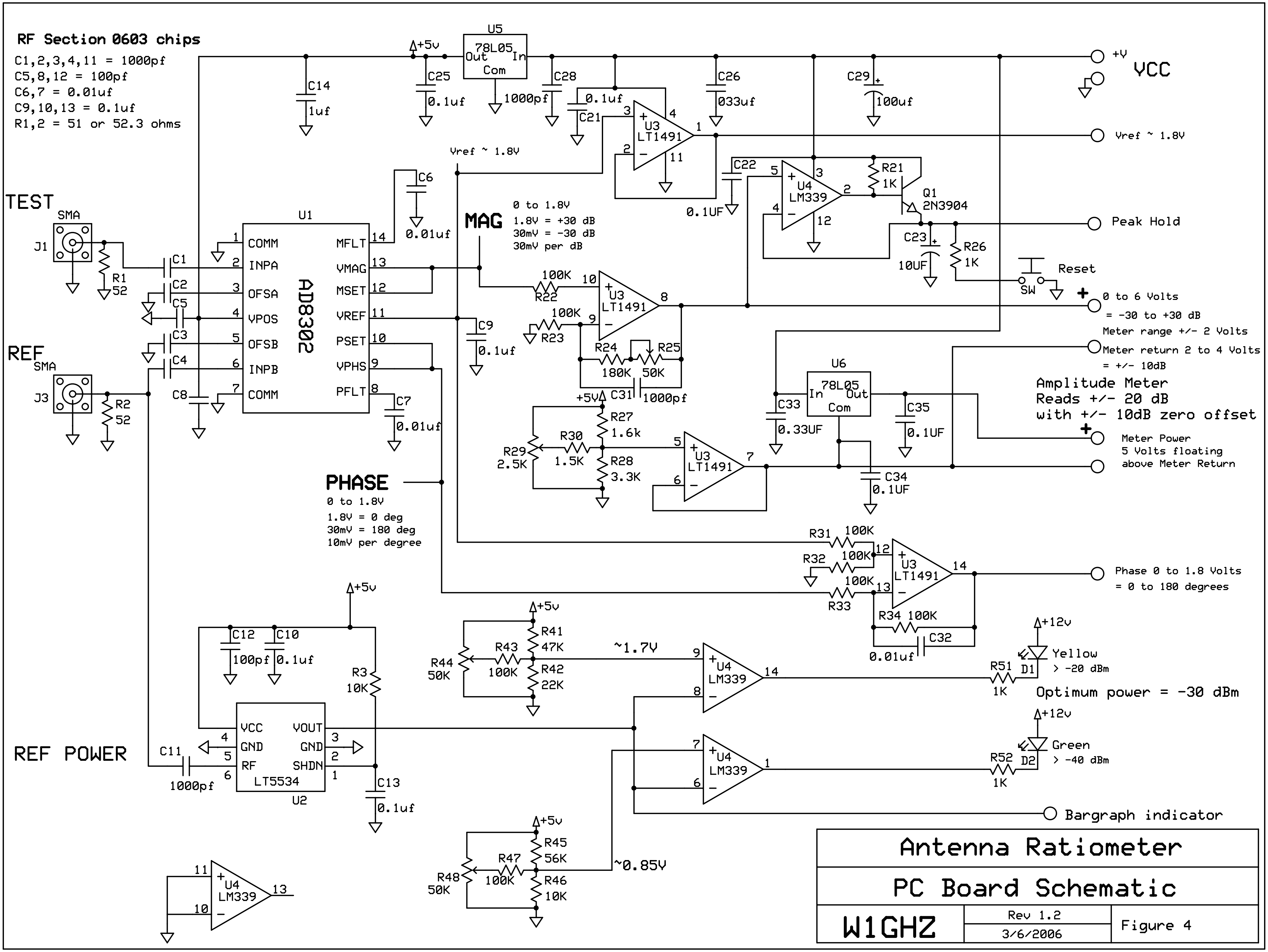
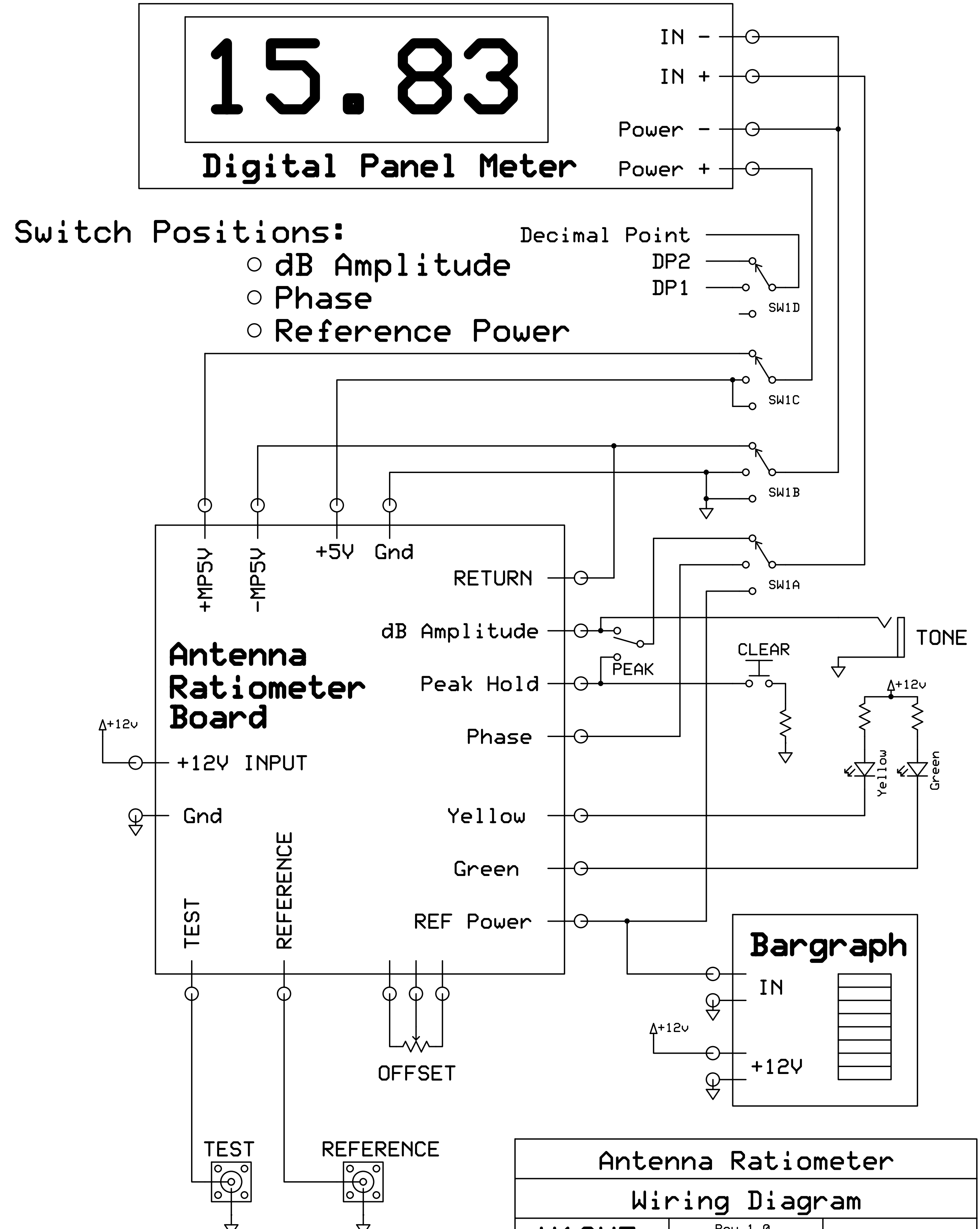


Figure 3



RF Section 0603 chips
C1,2,3,4,11 = 1000pf
C5,8,12 = 100pf
C6,7 = 0.01uf
C9,10,13 = 0.1uf
R1,2 = 51 or 52.3 ohms
TEST
SMA
J1
REF
SMA
J3
REF POWER
U1
AD8302
COMM
INPA
OFSA
VPOS
OFSB
INPB
COMM
MFLT
VMAG
MSET
VREF
PSET
VPHS
PFLT
U2
LT5534
VCC
GND
RF
VOUT
GND
SHDN
U5
78L05
U6
78L05
MAG
0 to 1.8V
1.8V = +30 dB
30mV = -30 dB
30mV per dB
PHASE
0 to 1.8V
1.8V = 0 deg
30mV = 180 deg
10mV per degree
Vref ~ 1.8V
VCC
+V
Peak Hold
Reset
0 to 6 Volts
= -30 to +30 dB
Meter range +/- 2 Volts
Meter return 2 to 4 Volts
= +/- 10dB
Amplitude Meter
Reads +/- 20 dB
with +/- 10dB zero offset
Meter Power
5 Volts floating
above Meter Return
Phase 0 to 1.8 Volts
= 0 to 180 degrees
~1.7V
~0.85V
Yellow
> -20 dBm
Green
> -40 dBm
Optimum power = -30 dBm
Bargraph indicator
Antenna Ratiometer
PC Board Schematic
W1GHZ
Rev 1.2
3/6/2006
Figure 4



Antenna Ratiometer

Wiring Diagram

W1GHZ	Rev 1.0 2/28/2005	Figure 5

The first real test of this meter was at the 2005 Eastern VHF/UHF Conference, where it was used for antenna measurements from 144 to 1296 MHz, and performed well while running from just a small battery. The test scheme is shown in Figure 6.

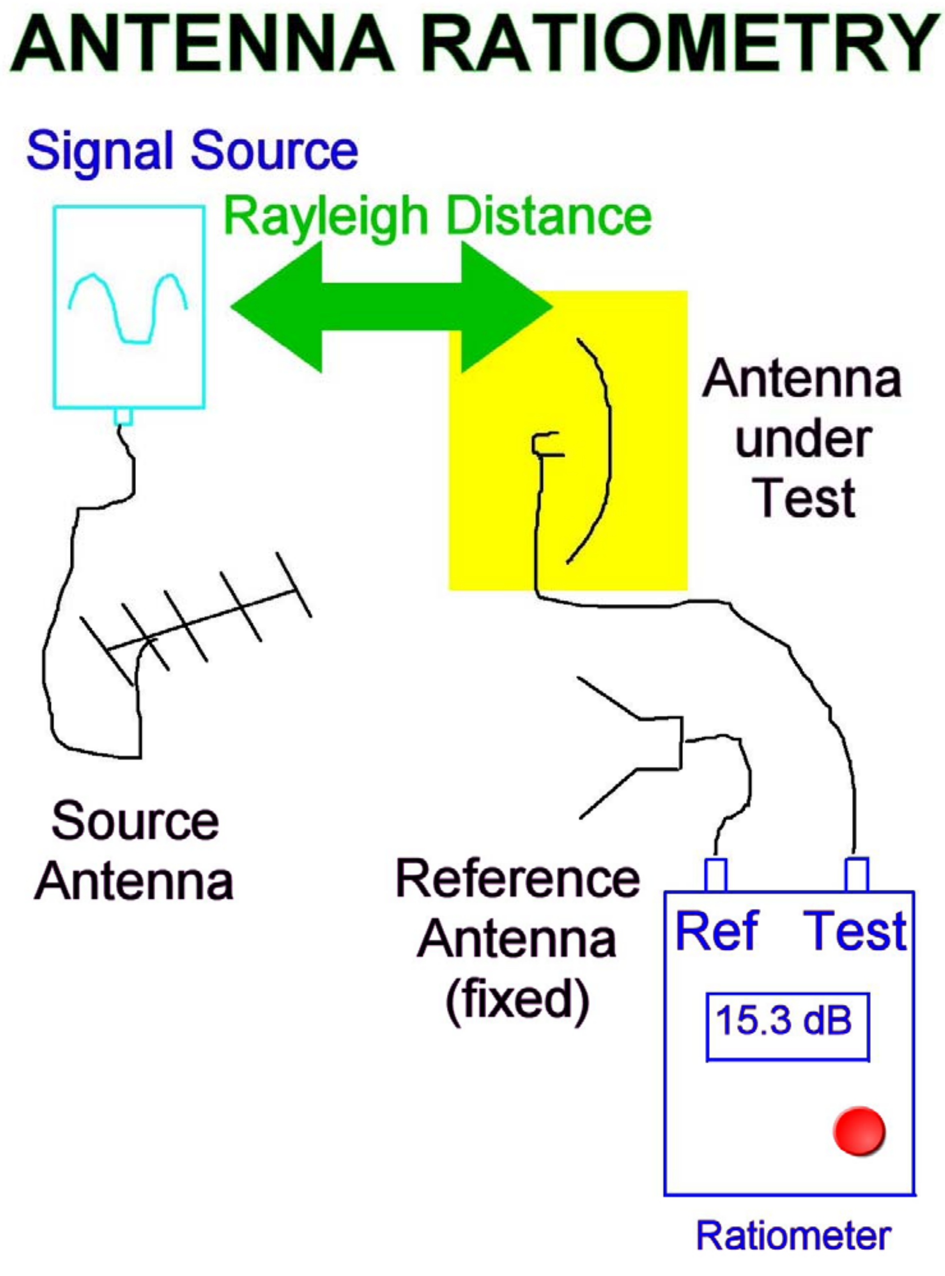


Figure 6

The ratiometer has a very broadband input; at this location, within line-of-sight of a cell tower, connecting antennas directly would be asking for trouble. On each band, filters were added in front of each input to eliminate unwanted signals. A photograph of the actual test setup for 432 MHz is shown in Figure 6a, with the beer-can filters plainly visible. I made these about 30 years ago, and they still work well. Unfortunately, most cans are now made of aluminum.

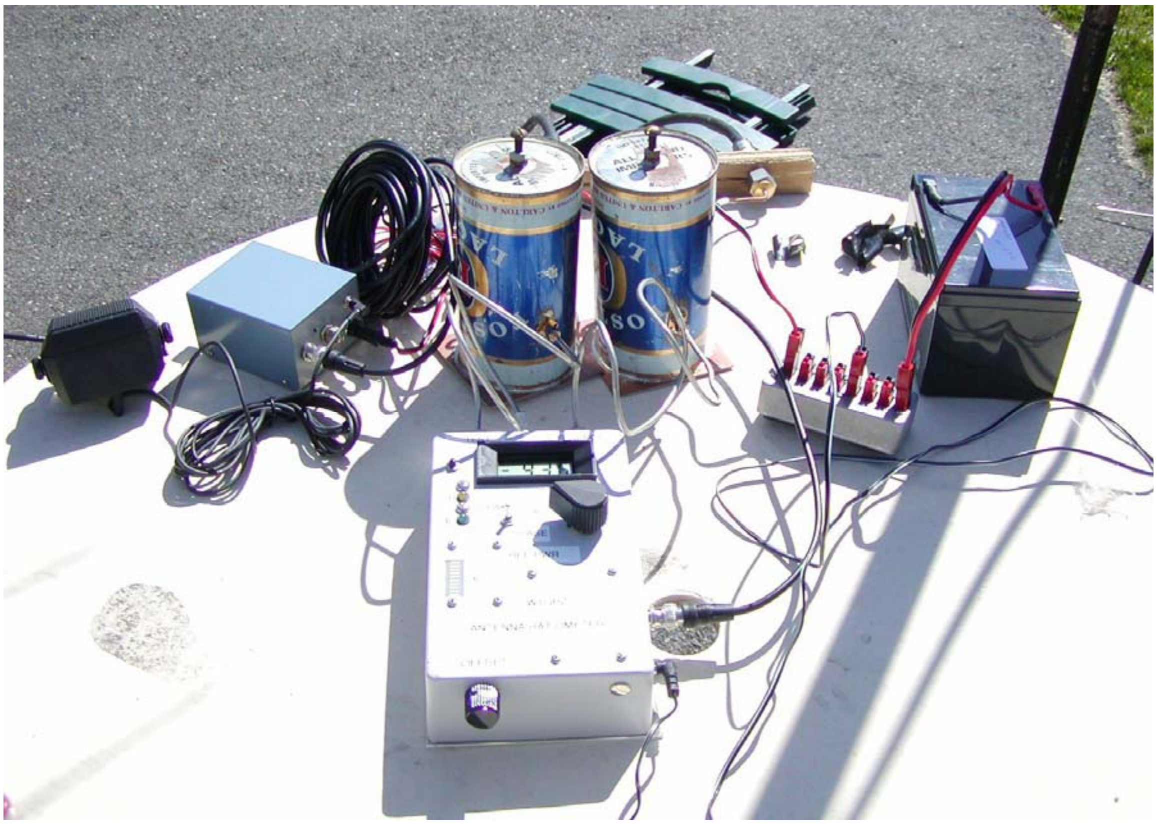


Figure 6a – Antenna measuring at 432 MHz

Antenna Range

Few of us have the luxury of a permanent antenna range, so we must set up a temporary range, sometimes at a remote location like a VHF conference. High antenna support structures are not usually available, so we must work near ground level and use a ground-reflection range, designing the range to account for ground reflection and to control it, as described by W2IMU[6,7]. The most common measurement goal is simply maximum antenna gain, typically with relatively high gain antennas. Other measurements, like pattern plots or phase-center, require more difficult antenna ranges.

For an antenna to operate with maximum gain, it must receive a wavefront with constant phase. Thus, the length of the antenna range is important — if it is too short, there will be significant phase difference over the aperture of the antenna being tested, resulting in low measured gain. The minimum range length to minimize this error is the Rayleigh distance:

$$\text{Rayleigh distance} = \frac{2\,D^2}{\lambda}$$

Longer range lengths will reduce the phase error and improve accuracy. A few trial calculations will show that miles of range can be required for large dishes. Fortunately, the Rayleigh distance for most transportable VHF and microwave antennas is less than 100 meters, so sites with sufficient open space are available. Note that this is the minimum length for gain measurement; for focusing adjustments, like the feed placement on a parabolic dish, a much longer distance, at least 50 times the Rayleigh distance, is required[8].

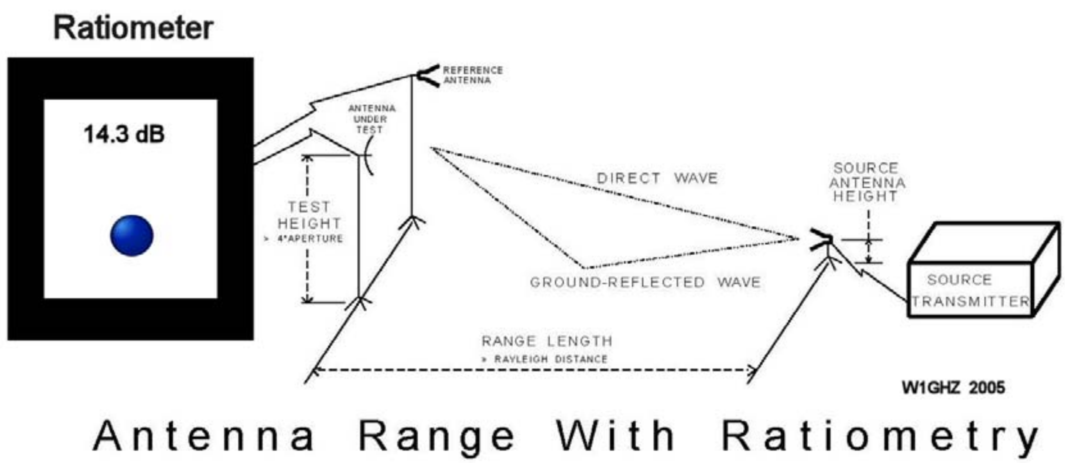


Figure 7

A ground-reflection range is sketched in Figure 7. In order for the phase error to be as low in the vertical plane as in the horizontal plane, the height of the antenna being measured must be at least four times its aperture diameter[9]:

$$\text{Test height} \geq 4 \bullet \text{Aperture diameter}$$

for example, 4 meters high for a one-meter dish. Most amateur antenna ranges have insufficient antenna height, and consequently have had trouble measuring higher-gain antennas accurately. For a Yagi-Uda antenna, the aperture diameter is roughly the same as the stacking distance; for a long boom 2-meter beam, the stacking distance might be 4 or 5 meters, so the antenna height must be 15 or 20 meters for accurate measurement.

The received energy should be at a maximum at the height of the antenna being measured. For a ground-reflection antenna range, this is controlled by the height of the source antenna:

$$\text{Source Height} = \frac{\lambda}{4} \bullet \frac{\text{Range length}}{\text{Measurement height}}$$

For example, for a 30-meter long range at 1296 MHz, if we desire the maximum energy at a height of 4 meters, then the source height is about 0.4 meters, or roughly knee-high.

So the source antenna is relatively low, while the receiving antennas, test and reference, need to be elevated – some sort of structure may be necessary.

Power

The good sensitivity of the homebrew Antenna Ratiometer means that only modest radiated power is needed for an antenna range. We can calculate the power using the pathloss for the range calculated by the Friis transmission formula[10]:

$$\text{Path loss}(dB) = 10\log\left(\frac{4\pi}{\lambda} \bullet \text{Pathlength}\right)^2 - \mathrm{G}_{\mathrm{TX}}(dBi) - \mathrm{G}_{\mathrm{RX}}(dBi)$$

For reasonable range lengths, less than 100 meters, the path loss for each VHF, UHF, and microwave band is shown in Table 1. I estimated the gain for reasonably-sized source and reference antennas for each band to calculate the source power required. Less than 1 watt is needed for a 100 meter range, which is long enough for a 0.75 meter dish at 24 GHz, a 1.2 meter dish at 10 GHz, and larger antennas at lower frequencies.

Gain Standard

In order to measure meaningful antenna gains, an antenna with known gain is required for comparison. Recall that all measurements are relative to a known standard. A dipole is useless as a standard — its broad pattern receives so many stray reflections that repeatable readings are nearly

Antenna Range Pathloss Calculator

W1GHZ 2005

Freq	λ	Path = 1 meter		Path = 10 meters		Path = 100 meters		Antenna Gain	
		Pathloss	Source Power	Pathloss	Source Power	Pathloss	Source Power	Source	Reference
144 MHz	2.08 m	**15.6** dB	**-26.4** dBm	**35.6** dB	**-6.4** dBm	**55.6** dB	**13.6** dBm	6	6 dBi
222 MHz	1.35 m	**19.4** dB	**-22.6** dBm	**39.4** dB	**-2.6** dBm	**59.4** dB	**17.4** dBm	6	6 dBi
432 MHz	0.69 m	**25.2** dB	**-20.8** dBm	**45.2** dB	**-0.8** dBm	**65.2** dB	**19.2** dBm	8	8 dBi
903 MHz	0.33 m	**31.6** dB	**-18.4** dBm	**51.6** dB	**1.6** dBm	**71.6** dB	**21.6** dBm	10	10 dBi
1296 MHz	0.23 m	**34.7** dB	**-15.3** dBm	**54.7** dB	**4.7** dBm	**74.7** dB	**24.7** dBm	10	10 dBi
2304 MHz	0.13 m	**39.7** dB	**-10.3** dBm	**59.7** dB	**9.7** dBm	**79.7** dB	**29.7** dBm	10	10 dBi
3456 MHz	0.09 m	**43.2** dB	**-12.8** dBm	**63.2** dB	**7.2** dBm	**83.2** dB	**27.2** dBm	12	14 dBi
5760 MHz	0.05 m	**47.7** dB	**-10.3** dBm	**67.7** dB	**9.7** dBm	**87.7** dB	**29.7** dBm	13	15 dBi
10368 MHz	0.03 m	**52.8** dB	**-11.2** dBm	**72.8** dB	**8.8** dBm	**92.8** dB	**28.8** dBm	17	17 dBi
24192 MHz	0.01 m	**60.1** dB	**-10.9** dBm	**80.1** dB	**9.1** dBm	**100.1** dB	**29.1** dBm	20	21 dBi

Table 1

impossible, and its gain is much lower than a 30+ dB dish that equipment accuracy is a problem; few instruments are accurate over a 30 dB (1000:1 power ratio) range. What is required is an antenna with a known gain, preferably gain of the same order of magnitude as the antennas to be measured. For VHF and UHF frequencies, the EIA standard gain antenna[6] is easily reproduced. At microwave frequencies, the gain of a horn antenna can be calculated quite accurately from the physical dimensions. The algorithm used in the HDL_ANT program[11] will be accurate within about 0.2 dB if good construction techniques are used. For even better accuracy, several companies make standard gain horns with good calibration data – these occasionally show up as surplus.

Range Measurements

Once the antenna range is designed and set up, it must be checked out before making actual measurements. This is best done with an antenna with a fairly broad pattern, like a medium-sized horn or Yagi, as the test antenna. First, the attenuators are adjusted for a convenient meter reading. Then the field uniformity is probed by moving the test antenna horizontally and vertically around the intended measurement point. The indicated gain should peak at the center and should not vary significantly over an area larger than any antenna to be tested; the variation should be less than one dB. Usually, the height of the source antenna needs to be adjusted to get the vertical peak at the desired receiving height. Finally, the test antenna is held stationary and calibrated attenuation steps are added in the test path to make sure the indicated gain changes by the amount of attenuation added.

Now the range is ready to make measurements. The standard gain antenna is inserted as the test antenna, aimed for maximum indication, and the attenuators and offset adjusted for a meter reading that will keep expected gains within the range of the meter. All gain measurements will be the difference from this standard reading added to the gain of the standard gain antenna. The standard gain antenna is replaced by an antenna to be tested, the new antenna aimed for maximum gain, and its indicated gain recorded. The difference between this indicated gain and the standard reading, added to the known gain of the standard gain antenna, is the gain of the test antenna. The reading with the standard gain antenna should be checked frequently to correct for instrumentation drift; ratiometry with the reference antenna corrects for other sources of drift.

Handheld Network Analyzer

It may have occurred to you already that the Antenna Ratiometer described above is really a modest network analyzer. It only has 40 dB of dynamic range, vs. >70 dB for computer-controlled lab models, but there aren't really many amateur measurements that require the full range or absolute accuracy of the fancy ones.

Accurate measurements over wide dynamic range require a lot of computer correction, which in turn requires precise frequency control for repeatability. Over a wide frequency range, an expensive synthesized signal generator is required – in surplus markets, this is often more valuable than the network analyzer and is sometimes removed and sold separately, making the rest of the network analyzer available at a reasonable price. For most ham work, fast swept-frequency measurements are not necessary and most measurements are in ham bands, so a much more affordable synthesized generator may be used. Mine is a Yaesu FT-817, as small and portable as the handheld network analyzer – it is also the IF rig for my microwave equipment. Of course, nearly any signal generator or sweeper is also usable; since only –30 dBm (*one microwatt*) is required, attenuation is the problem, not power.

For network analyzer use, I made a second version, shown in Figure 8, with two digital readouts to simultaneously display dB and phase. The digital panel meters are hardly more expensive than the rotary switch. The inside, Figure 9, is very similar to the other version. Two additional BNC connectors on the side bring out ratio and phase to an oscilloscope for swept measurements. An example, a swept filter response, is shown in Figure 13.

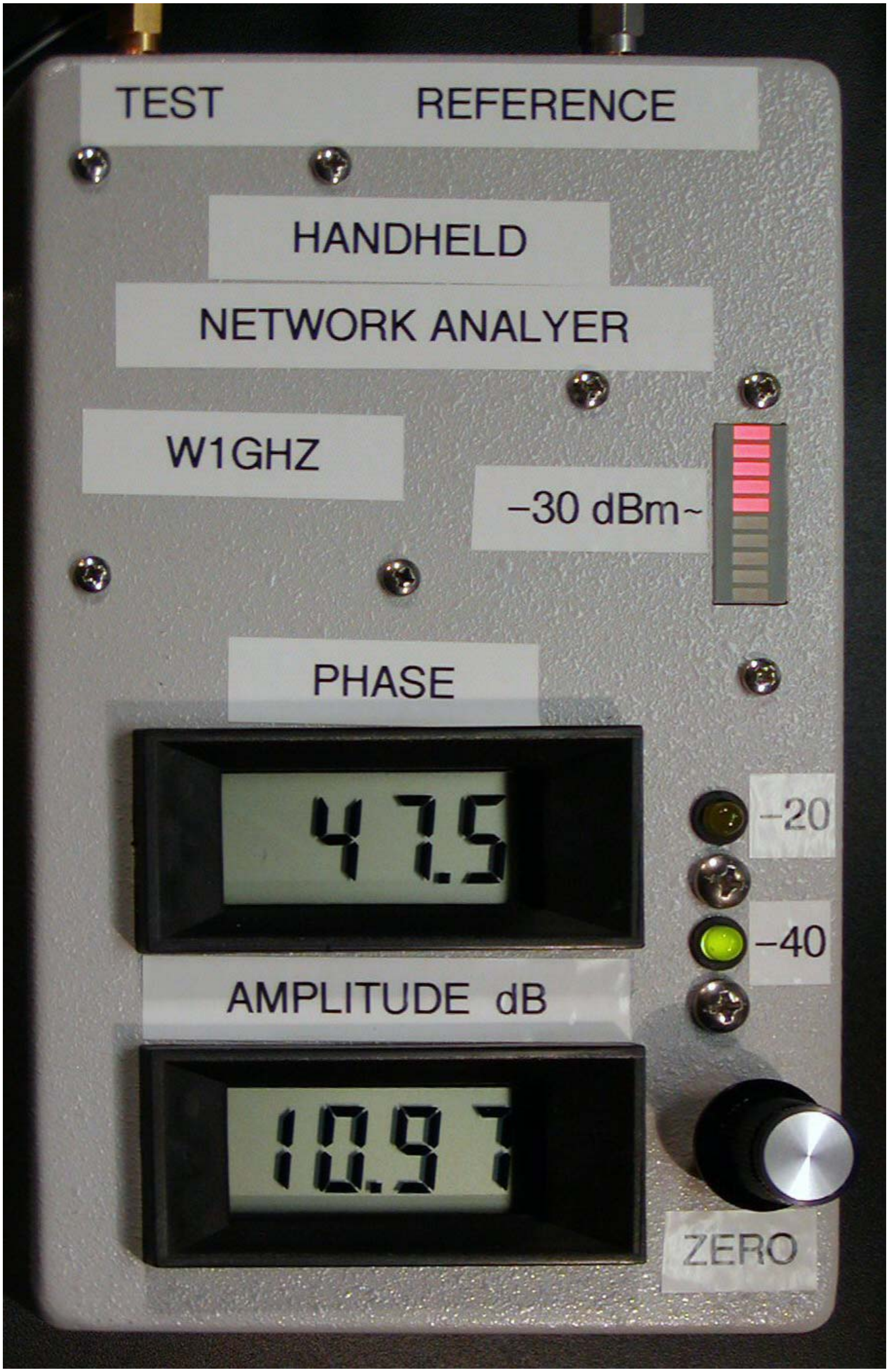

TEST
REFERENCE
HANDHELD
NETWORK ANALYER
W1GHZ
−30 dBm~
PHASE
47.5
−20
−40
AMPLITUDE dB
10.97
ZERO

Figure 8

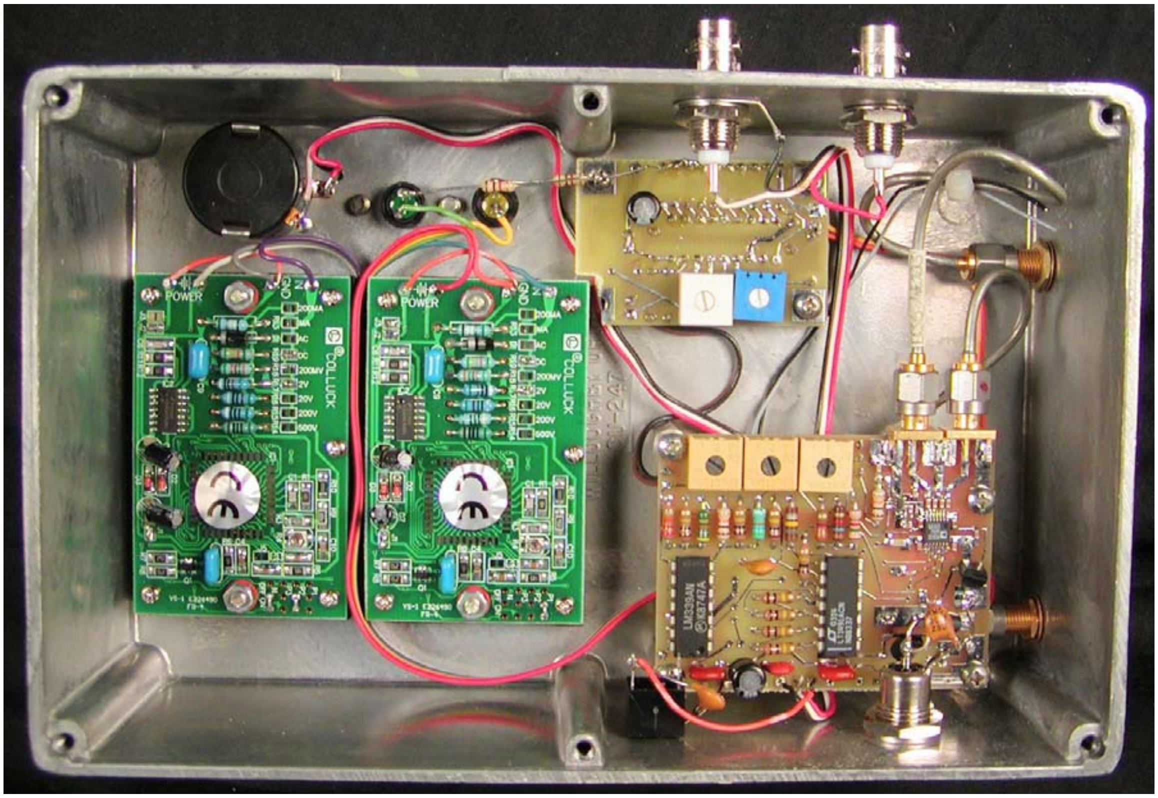


Figure 9

A network analyzer, or a ratiometer, has many uses besides antenna gain measurements. One of the more obvious, sketched in Figure 10, is to measure the VSWR (as Return Loss) of an antenna, or anything else. A directional coupler is used to sample the Forward power for the REFERENCE port and the Reflected power for the TEST port. Since we also measure phase, actual complex impedance may be found with a bit of calculation. Before connecting the antenna, the output is connected to a short circuit and the meter is set to zero.

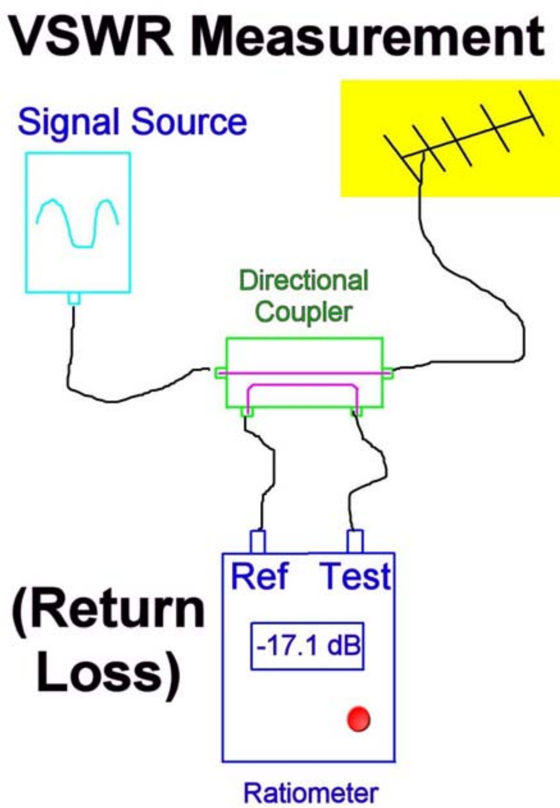


Figure 10

Since we can also measure phase as well as VSWR, it is possible to calculate complex impedance, using a Smith Chart or calculator. This can take the guesswork out of impedance matching.

A trick I use to check on the feedlines on my tower is to measure the Return Loss over a wider frequency range. Many antennas, like Yagis, are quite narrowband, so the Return Loss is high within the band but low at nearby frequencies. I look for the minimum return loss and guess that the antenna is reflecting nearly 100% of the power at that frequency, so that the measured Return Loss is just the loss of the coax going up to the antenna and back down. Thus, the coax loss is half the Return Loss. I keep track of the loss for each feedline and check them occasionally, so that I can catch any problems and not just wonder why I'm not hearing so well on one band.

The gain of an amplifier, Figure 11, or the loss of a passive component such as a cable or filter, Figure 12, may also be measured. A directional coupler is used to sample the input power, and an attenuator or a directional coupler for the output power. If the gain or loss exceeds 20 dB, a step attenuator is useful to bring the TEST reading within range. The added attenuation is added to the gain or loss. The meter is zeroed without the component in the circuit, with the output connected directly to the input.

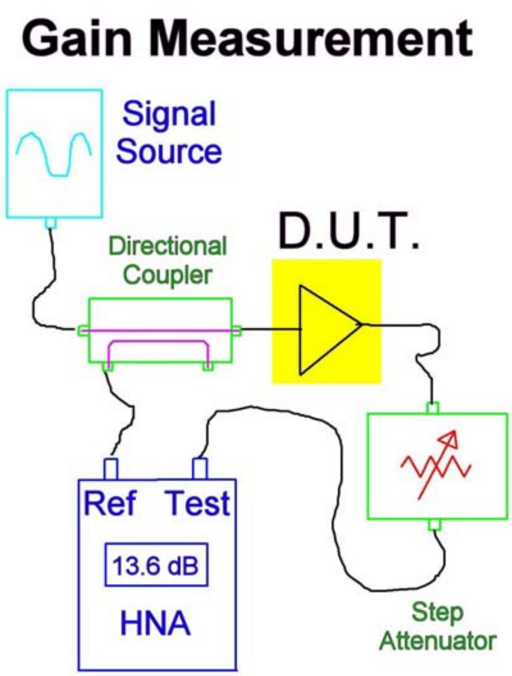


Figure 11

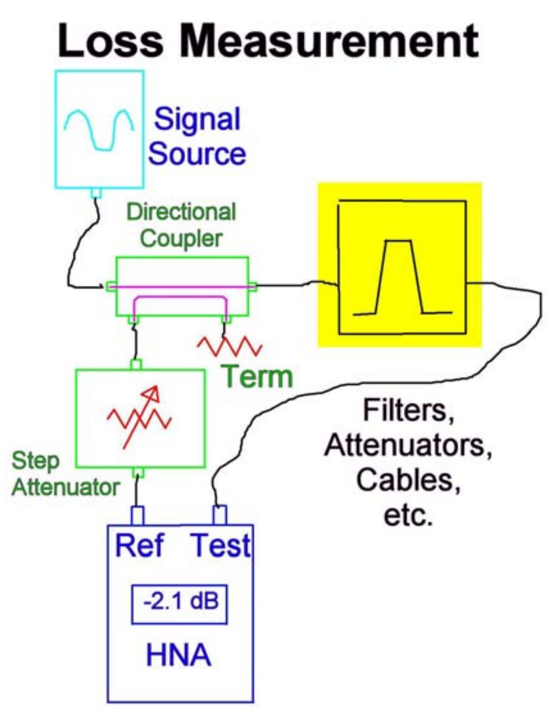


Figure 12

With a sweep signal source, we may sweep the frequency response of a circuit. Figure 13 shows an example: the swept frequency response of a Toko 144 MHz helical filter, swept from roughly 120 to 180 MHz. Both the amplitude and phase responses are shown. The amplitude is a smooth bandpass response, but the phase changes rapidly near the resonant frequency.

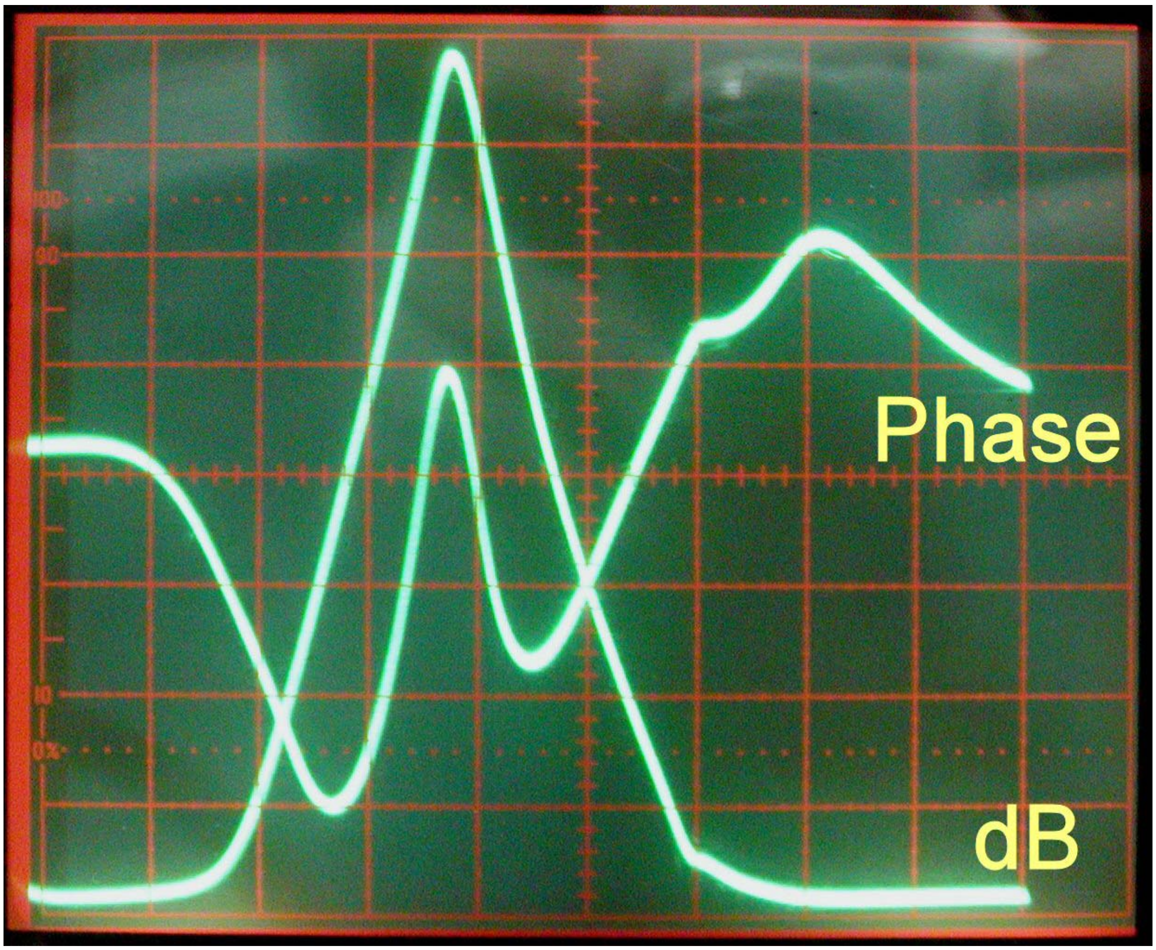


Figure 11

A common amateur problem is matching phasing lines for an antenna array. With the network analyzer, the lines may be directly matched for phase, comparing two at a time or comparing each one to a reference line, Figure 14. Note that the phase reading is compressed near zero and 180º, so it may be necessary to add a short length to one channel to read an intermediate value. I didn't include a zero set for phase, since relative measurements are usually sufficient.

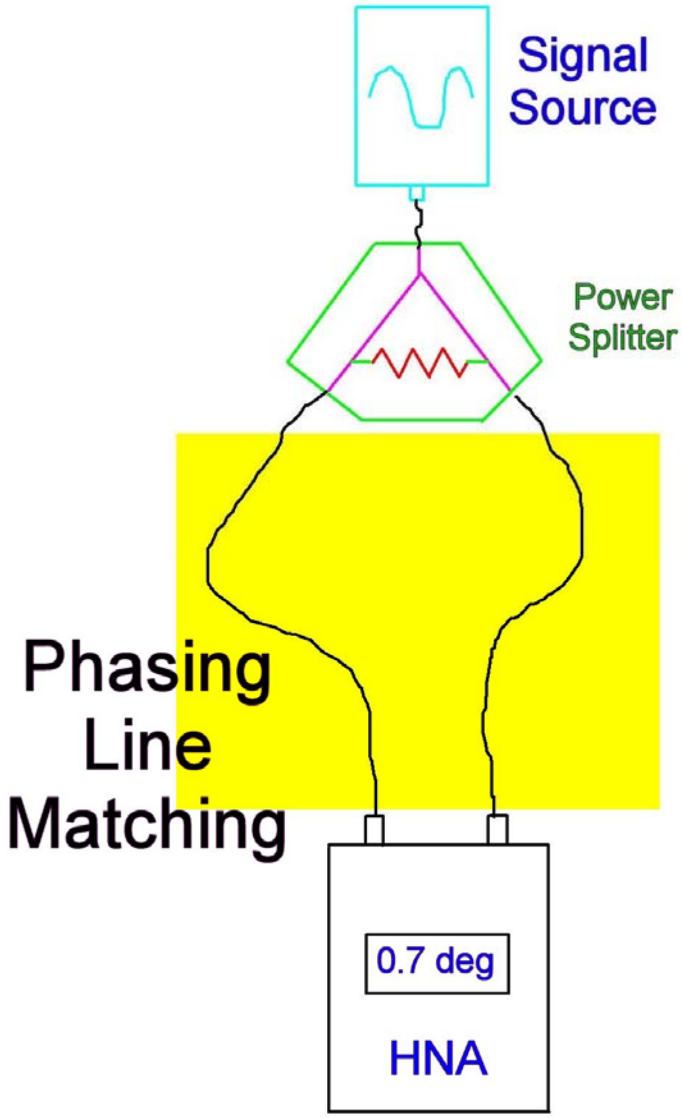


Figure 12

The AD8302 phase output is 0º to 180º. There is no sign output, so we don't know whether the phase difference is positive and negative. This doesn't matter when we are matching phases, but may require some thought for other measurements.

Another problem made easier is checking an antenna for circular polarization, or a hybrid coupler to get the 90º phasing required to generate circular polarization. There are two possible ways to make this measurement. One, in Figure 15, is to use a source antenna providing known good circular polarization, like a helix or septum feed: the two outputs should have equal amplitude. The other is to use a linear source antenna and look for 90º phase shift as the antenna is rotated. If the range has significant ground reflections, then the linear antenna should be left horizontal and the circular antenna rotated; with vertical polarization, ground reflections can have rapid phase shift.

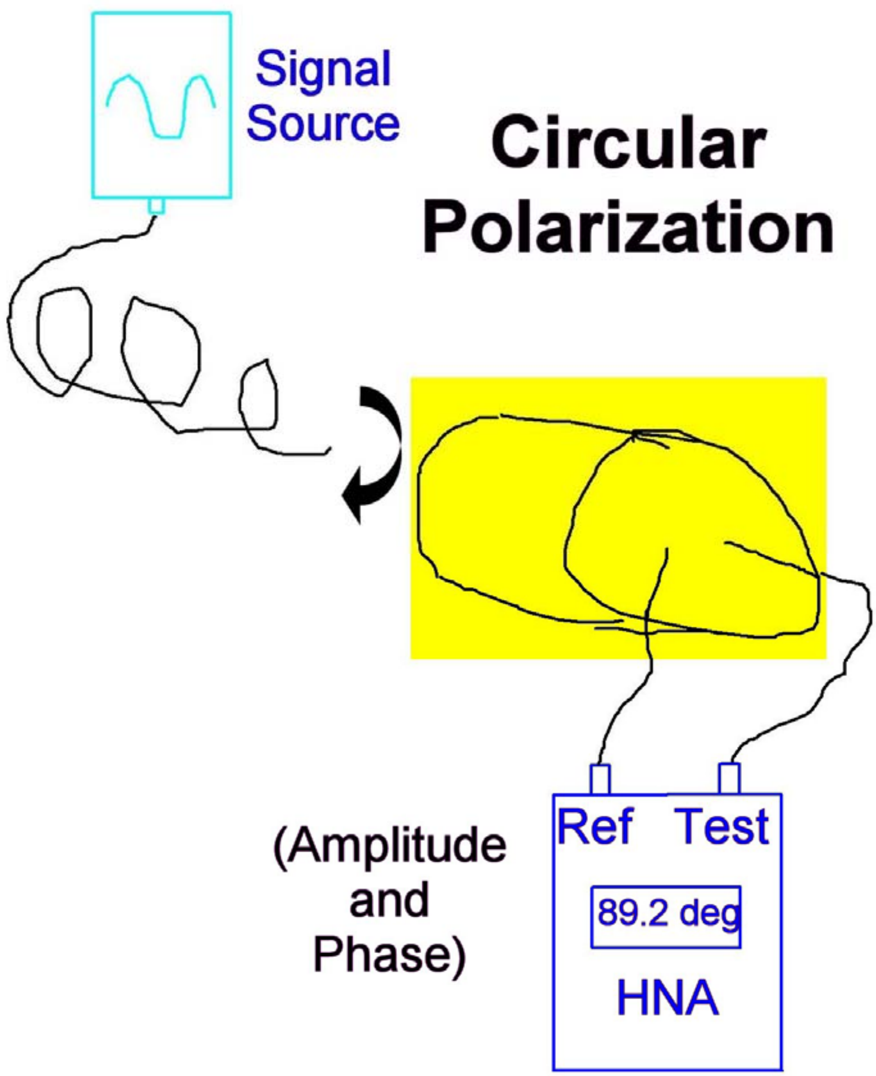


Figure 13

Higher Frequencies

The maximum frequency limit of this instrument is 2.7 GHz – that's the limit for the AD8302 chip. However, with a pair of mixers, one for each channel, and a local oscillator, we can convert both test and reference signals to frequencies below 2.7 GHz and thus extend the usable frequency as high as needed. The configuration is shown in Figure 15. The LO requirements are not too stringent – it just has to be within about 2.5 GHz of the measurement frequency, so that the IF outputs are suitable for the HNA. Only moderate stability is required, at least for amplitude measurements, so crystal control is not necessary. A DRO (Dielectric Resonator Oscillator) should be sufficient – many common TV LNBs for DSS have a DRO running at about 10.75 GHz, ideal for 10 GHz use.

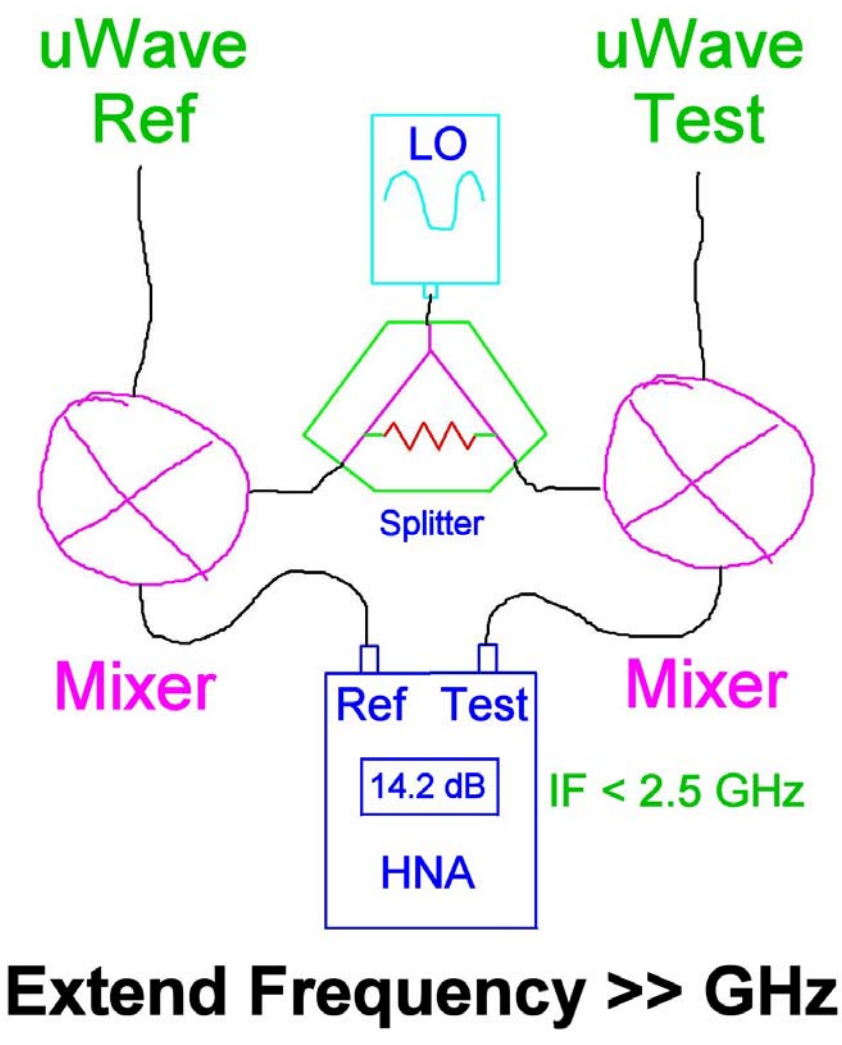


Figure 15

The mixers need not be matched. Since the dynamic range is only 40 dB, and the nominal output level is -30 dBm, almost any mixer would be suitable. The only precaution is that good isolation is required at the IF frequency between the two mixer outputs. Dual mixers on a single PCB may not have enough isolation.

With suitable mixers, we have a network analyzer usable at 24 GHz or higher. While mixers may be hard to come by, network analyzers for these frequencies are really rare and very expensive.

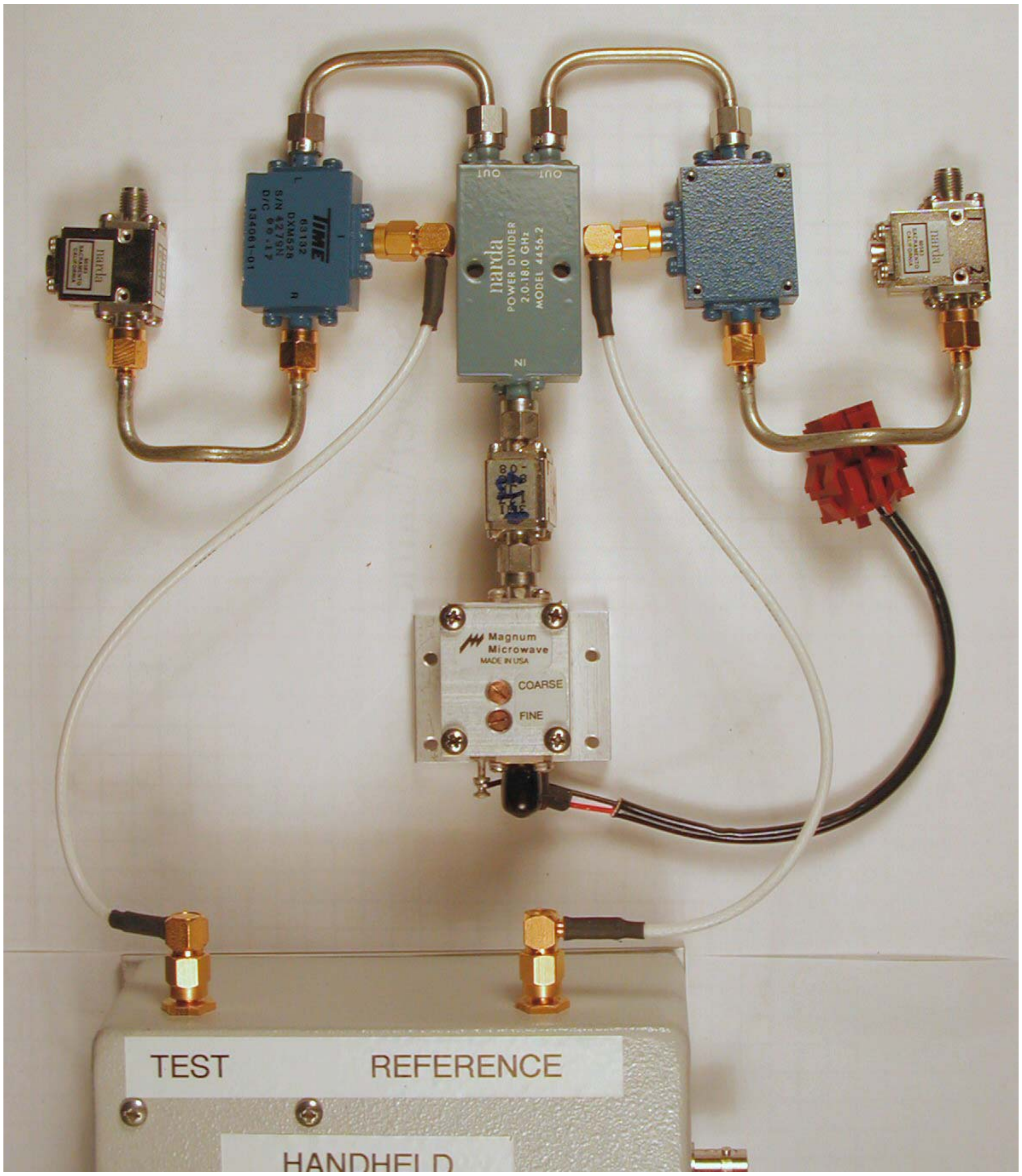


Figure 16 – 10 GHz Frequency Extender Breadboard

My breadboard of a 10 GHz frequency extender is shown in Figure 16 – try it out before doing metalwork! The LO is an 11 GHz DRO donated by W1AIM, and the rest of the components came from the junkbox, the results of scrounging at many hamfests. The DRO output is about +10.5 dBm, so it provides about +6 dBm to the mixers after the splitter – an ideal LO level for a mixer. The circuit is like Figure 15, with the addition of a couple of isolators to provide a good input match. Since the mixers have some conversion loss, a bit more signal is required for operation, about -23 dBm at 10 GHz to get a -30 dBm indication on the Handheld Network Analyzer. Thus, the mixer conversion loss is about 7 dB, as we would expect from a good mixer.

Summary

The combination of Antenna Ratiometer and Handheld Network Analyzer is a versatile and useful piece of test equipment, particularly for the VHF and microwave antenna experimenter. It should be possible to reproduce for under $100 – many hams spend far more than this on a piece of surplus test equipment – but it isn't absolutely necessary to have one. I'm amazed that some hams are able to accomplish impressive results with no test equipment at all, while others spend an impressive amount on test equipment that only gathers dust.

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